The geometric image of the world

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Abstract

Children recognize spontaneously geometric objects in the real world and refer to them with natural language expressions, but the common sense ideas are very different from the mathematical concepts. This article presents geometric activities, designed for pupils in the 11-14 age range, where Logo is integrated with other software (Cabri II Plus, a basic CAD, a graphics package) and with explorations into the real world. Our work has a double aim: to keep the children curiosity and interest alive, because without motivation there is not effective learning, and to provide a variety of environments suitable to help pupils transform their intuitive notions into more precise and formal ideas about geometric properties and mathematical concepts in general. The historical dimension played an important role in planning these experiences.

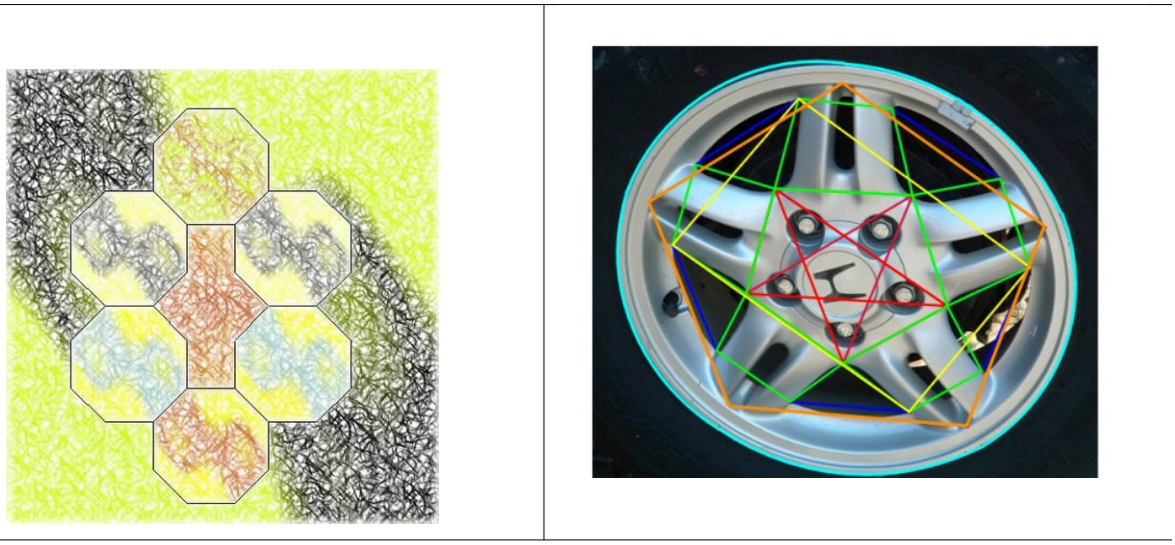

Figure 1. Left: Logo tessellation with graphic arts. Right: modelling of a rim with Cabri.

Keywords

geometry, history of mathematics, heuristic process, recursive thinking, motivation, dynamic geometry, Cabri, geometry and real world, Logo

Aims

- To build a sequence from a starting point to a foreseen result
- To compare the concept of angle as change in direction with the Euclidean concept of angle.
- To compare the trace left by the turtle with the Euclidean concept of edge
- To recognize the regular polygons as modulus in translations, rotations, symmetries (it is also possible to recall some modular arithmetic aspects)
- To compare the idea of circle as "the limit" of regular polygons with the Euclidean definition
- To be able to describe the position of an object by using different reference points (right or left related with….)
- To know and use properly the Cartesian reference system

- To be able to break down a problem in simpler steps, and then to rebuild the whole problem, by using a tree-graphs.
- To be able to distinguish variables from constants in a graphic project. This is a very important training that prepare pupils towards the analysis of algebraic problems.
- To use the structure: "if…then…otherwise" in elementary situations. This logical structure permit the control of recursive steps in geometrical problems (e.g. .spirals) or in numerical ones (e.g.series)

Framework

An effective teaching/learning of Geometry should take account of the following dimensions (2):

- the pedagogical one, raising questions like: 'How to motivate pupils?'', 'How to help them better understand, draw generalizations and construct meanings and competencies?', 'What kind of tools are more effective?,' What role can ICT play?'….
- the historical one, that is a source of suggestions for teachers, is motivating for pupils and show them the problematic nature of the process that led to our geometric knowledge.
- the epistemological one, because a reflection about the structure of the discipline and on its way of thinking allows to identify key concepts and procedures (e.g. the importance of the heuristic thought, the internal consistency, the study of patterns and relationships, the connection with the real world ..)

Historical overview and Logo

During the ancient Greek and Roman period

- Thales (642 B.C. – 548 B.C. ?): an angle inscribed in a semicircle is a right angle.
- Pythagoras (580 – 500 ?) and its theorem, the golden section of a segment, the figured numbers…
- The incommensurability: the diagonal of a square…
- The regular solids (cube, tetrahedron, octahedron, dodecahedron, icosahedron) and Plato…
- Euclid…
- Archimedes…
- Apollonius of Perga (262 – 190 ?): π (radian), ellipsis, parabola and hyperbola.

During and after the Renaissance

- The perspective in the arts: Leon Battista Alberti, Piero della Francesca…(around 1400)
- Cavalieri: method for calculating volumes
- Descartes and its reference system (1596 – 1650)
- Klein and the Erlangen Program (1872) about the geometrical transformations.
- Papert and Logo (1980)

Starting from this last step, we can come back to the Ancient Greek period, planning activities where pupils are guided to explore figures and relationships among them.

They can study, for example, how to build the procedure for drawing regular polygons. Stimulated to identify relationships between side number and angle measurement to solve this problem, they often discover by themselves the "Turtle theorem" (3). Pupils can also be encouraged to draw some regular polygons in the Euclidean way, using pencil, rule and compass or dynamic geometry software like Cabri.

Combining Cabri with Logo is particularly interesting not only because the use of different tools usually contributes in raising interest, but also and mainly because, if the children have the opportunity to observe shapes that gradually change from one into another, it become easier for them making conjectures on their geometric properties (see Erlangen Program).

Epistemology and Logo

The heuristic process

Programming with Logo is a good way to approach the heuristic-mathematical process, because pupils can develop their own projects, build procedures by themselves and correct them, guided by their own mistakes ('bricolage' method).
Making a procedure is a construction with an internal consistency, i.e. pupils cannot break the rules of the Logo commands or syntax. With Logo, pupils can see in real time and step by step if their procedure works/doesn't work and are stimulated to find corrections by reasoning both on the procedure and on the draw generated by the procedure. This is an excellent way of training children in inductive/deductive thinking and then of getting them used to the algorithmic method. Later, pupils will recognize and use the same way of thinking in the demonstrations of classical theorems (1; 3).

The recursive thought

An other strong point of Logo is the opportunity to break down a problem in simpler steps, and then to rebuild the whole problem (top-down/bottom-up method). Sometimes one of the simpler steps is the main problem itself (4): this is a recursive situation, like this Italian nursery-rhyme:
'Once upon a time there was a king, sitting on a sofa, who was telling to his servant: -Tell me a story!-, and the servant started telling - Once upon….'. Geometrical problems developed with Logo, like a spiral construction with the Fibonacci series, can help the pupils to use this form of thought.

The role of ICT in the geometric representation of the world

Geometry gives a complex representation of reality, because it 'perceive' and represent the reality in different ways and with different theories, sometimes contradictory between them, and because often moves far away from the reality.
The ICT help children grasp this complexity, because they give them the opportunity to analyze geometrical problems with different approaches, from the more intuitive to the more deductive. We found useful to integrate the use of Logo with other software, i.e. Cabri II Plus, a basic CAD software and a graphics package. While the CAD software has a deductive and abstract method of analyzing a geometric problem, like the 3D representation of a solid, Cabri offers a good environment to stimulate intuitive reasoning and modelling, for example when children use Cabri to find out what shape better fits the image of a real object (5; 8; 10).
Graphics packages can be used to make more appealing Logo drawing and to animate them, in order to capture the interest of pupils, who like a lot this kind of activities.

For an exemplification of contents and approaches for 11-14 year old pupils see table n.1

Table 1. Examples of contents in different learning environment

	Pencil and paper	**Logo**	**Cabri II Plus**	**3D CAD and Graphics package**
Euclidean geometry	Drawing shapes and analysing their elements	Building a procedure to draw any regular polygon (external angles and 'Turtle theorem')	Analysing geometric properties by manipulating dynamic shapes (search for figures "dragging-resistant")	Analysing 3D shapes by representing them through 2D projections and cross sections, including plan and elevation…
Analytical geometry	Representing points and figures in a Cartesian plane	Envelopes	Modelling the reality: searching a parabola, hyperbole, ellipse, circle in real objects	
Geometrical transformations	Rotations, symmetries, translations…	Regular polygons and tessellations	Using vectors, symmetry axis, homothetic ratio to transform given figure and to realize creative drawing	Moving 3D shapes on different axis
Reasoning and describing	Explaining and justifying inferences and deductions using mathematical reasoning	Exploring connections in geometry; pose conditional constraints of the type 'If ... then ...'; and ask questions like 'What if ...?' or 'Why?'	Showing step by step deduction in solving a geometrical problem.	Animating and describing 3D shapes.

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